Some Characteristic Function of Fuzzy Matrix

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Abstract

Many application of matrices in both engineering and science utilize Eigen values and Eigen vectors. Control theory, vibration analysis, electric circuits, advanced dynamics and quantum mechanics are the few of the applications area. In this paper, first time we introduced the Eigen values and Eigen vectors of fuzzy matrix.

Introduction about Eigen values, Eigen vectors and fuzzy matrix. Proposed definitions of Eigen values, Eigen vectors, the application of proposed Eigen values and Eigen vectors of fuzzy matrix.

Computation of fuzzy Eigen values and fuzzy Eigen vectors of a fuzzy matrix is a challenging problem. Determining the maximal and minimal symmetric solution can help to find the Eigen values. So, we try to compute these Eigen values by determining the maximal and minimal symmetric solution of the fully fuzzy linear system $\tilde{A}\tilde{X} = \lambda\tilde{X}$.

Key words: Eigen value, Eigen vectors, fuzzy matrix and symmetric solution.

Introduction

The Eigen value problem is a problem of considerable theoretical interest and wide-ranging application. For example, this problem is crucial in solving systems of differential equations, analyzing population growth models, and calculating powers of matrices (in order to define the exponential matrix). Other areas such as physics, sociology, biology, economics and statistics have focused considerable attention on “Eigen values” and “Eigen Vectors” – their applications and their computations.

The basic concept of the fuzzy matrix theory is very simple and can be applied to social and natural situations. A branch of fuzzy matrix theory uses algorithms and algebra to analyze. It is used by social scientists to analyze interaction and can be used to complement analyses carried out using game theory or other analytical tools.

The problem of finding Eigen values arises in a wide variety of practical applications. It arises in almost all branches of science and engineering.

Proposed definitions and examples:

The proposed Characteristic equations of fuzzy matrix, polynomial equations of fuzzy matrix, working rule to find characteristic equation of fuzzy matrix, fuzzy Eigen values and Eigen vectors, properties of fuzzy Eigen values and Eigen vectors are presented as follows:

Characteristic Equation of fuzzy matrix

Consider the linear transformation $Y = A_F X$.

In general, this transformation transforms a column vector

$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ into the another column vector $Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$

By means of the square fuzzy matrix A_F where

$$A_F = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

If a vector X is transformed into a scalar multiple of the same vector. i.e., X is transformed into λX, then $Y = \lambda X = A_F X$.

$Y = A_F X$

$=> \lambda X = A_F X$

$A_F X - \lambda X = 0$

i.e., wher I is the unit matrix of order 'n'.

$A_F X - \lambda I X = 0$

$(A_F - \lambda I) X = 0$

$$\left[\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ . & \cdots & \cdots & \cdots \\ . & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

$$\left[\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} - \begin{bmatrix} \lambda & 0 & \cdots & 0 \\ 0 & \lambda & \cdots & 0 \\ . & \cdots & \cdots & \cdots \\ . & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \lambda \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

$$\left[\begin{bmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

$$(a_{11}-\lambda)\, x_1 + a_{12}x_2 + \ldots + a_{1n}\, x_n = 0$$

i.e.,

$$\begin{array}{l} a_{21}\, x_1 + (a_{22} - \lambda)\, x_2 + \ldots + a_{2n}x_n = 0 \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + (a_{nn} - \lambda)\, x_n = 0 \end{array} \quad \text{-- > (2)}$$

This system of equations will have a non-trivial solutions, if

$|A_F - \lambda I| = 0.$

i.e., $$\begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{vmatrix} = 0 \quad \text{-- (3)}$$

The equation $|A_F - \lambda I| = 0$ or equation (3) is said to be the characteristic equation of the transformation or the characteristic equation of the matrix A.

Solving $|A_F - \lambda I| = 0$, we get n roots forλ, these roots are called the characteristic roots or Eigen values of the matrix A_F.

Corresponding to each of value of λ, the equation $A_FX = \lambda X$ has a non-zero solution vector X. Let X_r, be the non-zero vector satisfying $A_FX = \lambda X$.

When $\lambda = \lambda_r$, X_r is said to be the latent vector or Eigen vector of a matrix A_F corresponding to λ_r.

Characteristic Polynomial of Fuzzy Matrix

The determinant $|A_F - \lambda I|$ when expanded will give a polynomial, which we call as characteristic polynomial of fuzzy matrix A_F.

Eigen Values and Eigen Vectors of a Fuzzy Matrix

Fuzzy Eigen values or proper values or Latent roots or characteristic roots

Let $A_F = [a_{ij}]$ be a square matrix.

The characteristic equation of A_F is $|A_F - \lambda I| = 0$.

The roots of the characteristic equation are called Fuzzy Eigen values of A_F.

Eigen vectors or Latent vector:

Let $A_F = [a_{ij}]$ be a fuzzy square matrix. If there exists a non-zero vector X = $\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$. Such that $A_FX = \lambda X$, then the vector X is called Eigenvector of A_F corresponding to the fuzzy Eigen values

Note:

1. Corresponding to n distinct Eigen values, we get n independent Eigen vectors.
2. If two or more Fuzzy Eigen values are equal, then it may or may not be possible to get linearly independent Eigenvectors corresponding the repeated Fuzzy Eigen Values.
3. If X_i is a solution for an Eigen Value λ_I, then it follows from $(A_F-\lambda I)$ X=0 that CX_i is also a solution, where C is an arbitrary constant. Thus, the Eigenvector corresponding to a fuzzy Eigen value is not unique but may be any one of the vectors CX.
4. Algebraic multiplicity of a Fuzzy Eigen value λ is the order of the fuzzy Eigen value as a root of the characteristic polynomial (i.e., if λ is a double root then algebraic multiplicity is 2).
5. Geometric multiplicity of λ is the number of linearly independent Eigenvectors corresponding toλ.

Working Rule to find Eigen values and Eigen Vectors

Step: 1

Find the characteristic equation $|A_F - \lambda I| = 0$.

Step: 2

Solving the characteristic equation, we get characteristic roots. They are called Fuzzy Eigen values.

Step: 3

To find the Eigen Vectors, solve $(A_F - \lambda I)X = 0$ for the different values of λ.

Non-Symmetric matrix

If a fuzzy square matrix A_F is non-symmetric, then $A_F \neq A^T{}_F$.

Note:

1. In a non-symmetric fuzzy matrix, the fuzzy Eigen values are non-repeated then we get linearly independent set of Eigen vectors.
2. In a non-symmetric fuzzy matrix the fuzzy values are repeated and then we may or may not be possible to get linearly independent Eigen vectors. If we form linearly independent sets of Eigen vectors, then diagonalisation is possible through similarly transformation.

Symmetric matrix:

If a fuzzy square matrix A_F is symmetric, then $A_F = A_F^T$.

Note:

i) In a symmetric fuzzy matrix the fuzzy Eigen values are non-repeated, and then we get a linearly independent and pair wise orthogonal sets of Eigenvectors.
ii) In a symmetric fuzzy matrix the fuzzy Eigen values are repeated, and then we may or may not be possible to get linearly independent and pair wise orthogonal sets of Eigen vectors. If we form linearly independent and pair wise orthogonal sets of Eigen vectors, the diagonalisation is possible through orthogonal transformation.

Conclusion

For solving fully fuzzy linear systems (FFLS) to determine pairs of Eigen values and Eigen vectors of a fuzzy matrix by using the maximal and minimal symmetric spreads. To the best of our knowledge, this is the first paper in the literature devoted to the study of such kind of problems.

In this paper, derived the properties of Eigen values and Eigen vectors for the fuzzy matrix, fuzzy matrix is vast area and the application of Eigen values and Eigen vectors of fuzzy matrix are Heat transfer equations, Control theory, vibration analysis, electric circuits, advanced dynamics and quantum mechanics. Moreover the Eigen values of fuzzy matrix satisfies the properties of Eigen values and Eigen vectors is the main objective of this research paper.

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